Bury College

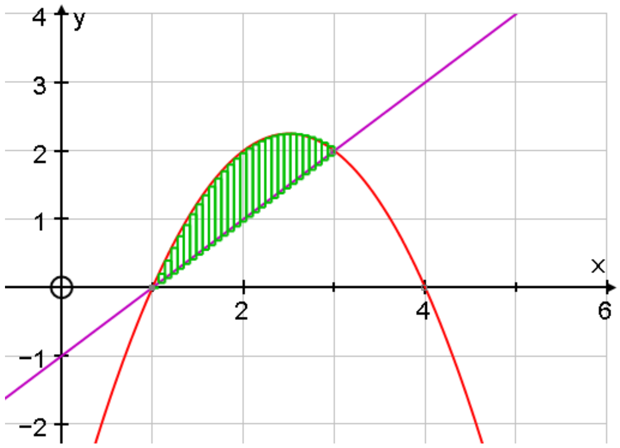
Mathematics Department

Introduction to A level Maths

BASIC ALGEBRAIC SKILLS BOOKLET

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$y - y_1 = m(x - x_1)$$



$$(x-a)^2 + (y-b)^2 = r^2$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$A = \frac{1}{2}r^2\theta$$

$$x^{\frac{p}{r}} = \left(\sqrt[r]{x}\right)^p$$

$$S_\infty = \frac{a}{1-r}$$

INTRODUCTION TO ADVANCED LEVEL MATHS

Welcome to the study of Advanced Mathematics at Bury College.

When your lessons begin, your mathematics teacher(s) will provide you with more information regarding the structure of the course. Some information will be in the form of handouts, booklets or directions to relevant sections of the College VLE, but other important information will be related to you directly in the classroom so remember to listen carefully from the outset and do not hesitate to **ask questions if you are unsure about anything**.

This booklet is designed to be a SHORT REVISION AID IN ALGEBRA for all students starting out in A Level mathematics. It covers the main algebraic principles and techniques from the GCSE higher level mathematics syllabus and many of you should have no problems working through the booklet in this time leading up to your course.

The importance of algebra in advanced level mathematics cannot be stressed too highly. It is essential that you begin your maths course with good GCSE algebraic skills and that you are willing to work hard on developing more advanced algebraic skills in order to cope with the demands of the course.

GENERAL ADVICE about using this booklet:

- try to keep your algebraic work **quite small** because working space is very limited in the booklet – this may be very difficult at first; you will have plenty of working space in the exam papers though!
- it is a good idea to use a pencil/eraser to keep your work tidy
- it is **very important** that you try to set out your solutions using **clear steps** – look carefully at how the solutions to the examples have been set out and try to set out your solutions in a similar way
- **keep checking your answers with those given at the back of the booklet**
- do not be afraid to **ask for help** when you are stuck or even if you just want your teacher to check that you are setting out your solutions 'correctly'

CONTENTS

Chapter 1: REMOVING BRACKETS

To remove (or expand) a single bracket, we multiply the term on the outside of the bracket by each term inside the bracket – paying particular attention to signs and the rules of indices.

E1) $3(x + 2y) = 3x + 3(2y) = 3x + 6y$

E2) $-2(2x - 3) = (-2)(2x) + (-2)(-3)$
$= -4x + 6$ **or** $6 - 4x$

E3) $5ab^2(2ab - c) = 5ab^2(2ab) + 5ab^2(-c)$
$= 10a^2b^3 - 5ab^2c$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including:
- the 'smiley face' method,
- the FOIL procedure (Firsts Outers Inners Lasts), or
- using a multiplication grid.

Examples:

E1) $(x + 1)(x + 2) = x(x + 2) + 1(x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$

or

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

or

	x	1
x	x^2	x
2	$2x$	2

$(x + 1)(x + 2) = x^2 + 2x + x + 2$
$= x^2 + 3x + 2$

E2) $(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$
$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

or

$(x - 2)(2x + 3) = 2x^2 + 3x - 4x - 6 = 2x^2 - x - 6$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

Exercise 1A Multiply out the following brackets and simplify if possible:

1) $7(4x + 5)$

2) $-3(5x - 7)$

3) $5a - 4(3a - 1)$

4) $4y + y(2 + 3y)$

5) $-3x - (x + 4)$

6) $5(2x - 1) - (3x - 4)$

7) $(x + 2)(x + 3)$

8) $(t - 5)(t - 2)$

9) $(2x + 3y)(3x - 4y)$

10) $4(x - 2)(x + 3)$

11) $(2y - 1)(2y + 1)$

12) $(3 + 5x)(4 - x)$

Two Special Cases

Perfect Square:	**Difference of two squares:**
$\boldsymbol{(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2}$	$\boldsymbol{(x - a)(x + a) = x^2 - a^2}$
$(x + 5)^2 = (x + 5)(x + 5) = x^2 + 10x + 25$	$(x - 3)(x + 3) = x^2 - 3^2$
$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$	$= x^2 - 9$

Exercise 1B Multiply out the brackets and simplify your results:

1) $(x - 1)^2$

2) $(3x + 5)^2$

3) $(7x - 2)^2$

4) $(x + 2)(x - 2)$

5) $(3x + 1)(3x - 1)$

6) $(5y - 3)(5y + 3)$

Chapter 2: SOLVING LINEAR EQUATIONS

The simplest type of linear equation will only involve one unknown (usually x) but this may appear more than once in the equation. The equation will also be constructed using some numbers (possibly including fractions/decimals), an equals sign, some mathematical operations and maybe some brackets. However, linear equations will not contain terms involving powers and roots such as x^2 and $\sqrt{x}$, etc.

There are two main methods of solving linear equations:

- the balance method,
- the 'change side/change sign' method.

These two methods are basically the same but your teacher and textbook will probably use the 'change side/change sign' method because it involves writing slightly less algebra.

To solve a linear equation (in x) you would normally follow these basic steps:

- multiply out all the brackets,
- collect all the terms which involve x on one side of the equals sign and all the other terms on the opposite side of the equals sign,
- simplify both sides by 'collecting together like terms',
- work towards a solution in the form ' $x = \ldots$' or '$\ldots = x$ ' ; this usually involves a final division operation.

Example 1: Solve the equation $64 - 3x = 25$

Solution: $64 - 3x = 25$

$$64 - 25 = 3x$$

$$39 = 3x$$

$$13 = x$$

Example 2: Solve the equation $6x + 7 = 5 - 2x$

Solution: $6x + 7 = 5 - 2x$

$$6x + 2x = 5 - 7$$

$$8x = -2$$

$$x = -{}^{2}/_{8} = -{}^{1}/_{4}$$

Example 3: Solve the equation $0.5x - 0.8 = 0.3x + 1$

Solution: $0.5x - 0.8 = 0.3x + 1$

$$0.5x - 0.3x = 1 + 0.8$$

$$0.2x = 1.8$$

$$x = {}^{1.8}/_{0.2} = {}^{18}/_{2} = 9$$

Exercise 2A Solve the following equations, showing each step in your working:

1) $2x + 5 = 19$

2) $5x - 2 = 13$

3) $11 - 4x = 5$

4) $5 - 7x = -9$

5) $11 + 3x = 8 - 2x$

6) $7x + 2 = 4x - 5$

7) $11 - x = 1 + x$

8) $1 - x = 1 + x$

9) $13x + 31 = 0$

10) $x - 2x + 3x - 4x = 6$

11) $4p - 7 = 0.5p$

12) $4\pi x = 18\pi - 2\pi x$

13) $3x + 2 - x = -5 + 2x + 7$ [This 'equation' will probably lead to an unexpected result]

14) $4x - 1 - x = 3x + 3$ [This 'equation' may lead to a result which is NONSENSE!]

Example 4 Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Solution:

$$2(3x - 2) = 20 - 3(x + 2)$$
$$6x - 4 = 20 - 3x - 6$$
$$6x + 3x = 20 - 6 + 4$$
$$9x = 18$$
$$x = 2$$

Exercise 2B Solve the following equations.

1) $5(2x - 4) = 4$

2) $4(2 - x) = 3(x - 9)$

3) $8 - (x + 3) = 4$

4) $14 - 3(2x + 3) = 2$

SOLVING EQUATIONS CONTAINING FRACTIONS

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

Example 5: Solve $\frac{y}{2} + 5 = 11$

Solution:

$$\frac{y}{2} + 5 = 11$$
$$y + 10 = 22$$
$$y = 12$$

Example 6: Solve $\frac{1}{3}(2x + 1) = 5$

Solution:

$$\frac{1}{3}(2x + 1) = 5$$
$$2x + 1 = 15$$
$$2x = 14$$
$$x = 7$$

When an equation contains two or more fractions, you need to **multiply each term** by the lowest common denominator, i.e. the **lowest common multiple (LCM) of all the denominators**.

Example 7: Solve $\frac{x+1}{4} + \frac{x+2}{5} = 2$

Solution:

$$\frac{20(x+1)}{4} + \frac{20(x+2)}{5} = 40$$
$$5(x+1) + 4(x+2) = 40$$
$$5x + 5 + 4x + 8 = 40$$
$$9x + 13 = 40$$
$$9x = 27$$
$$x = 3$$

Example 8: Solve $x + \frac{x-2}{4} = 2 - \frac{3-5x}{6}$

Solution:

$$12x + \frac{12(x-2)}{4} = 24 - \frac{12(3-5x)}{6}$$
$$12x + 3(x-2) = 24 - 2(3-5x)$$
$$12x + 3x - 6 = 24 - 6 + 10x$$
$$15x - 6 = 18 + 10x$$
$$5x = 24$$
$$x = 4.8$$

<u>Exercise 2C</u> Solve these equations:

1) $\frac{1}{2}(x+3)=5$

2) $\frac{2x}{3}-1=\frac{x}{3}+4$

3) $\frac{y}{4}+3=5-\frac{y}{3}$

4) $\frac{x-2}{7}=2+\frac{3-x}{14}$

5) $\frac{7x-1}{2}=13-x$

6) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

7) $2x+\frac{x-1}{2}=\frac{5x+3}{3}$

8) $2-\frac{5}{x}=\frac{10}{x}-1$

FORMING AND SOLVING (SIMPLE LINEAR) EQUATIONS

Example 9: Find three consecutive numbers such that their sum is 96.

Solution: Let the first number be n, then the second is $n + 1$ and the third is $n + 2$.
Therefore,

$$\begin{aligned} n + (n + 1) + (n + 2) &= 96 \\ 3n + 3 &= 96 \\ 3n &= 93 \\ n &= 31 \end{aligned}$$

So the numbers are 31, 32 and 33.

Exercise 2D: **Use algebra** to solve the following problems:

1) Find four consecutive **even numbers** such that their sum is 84.

2) In the triangle ΔABC, B is 4° larger than C, and A is twice the size of B.
Solve for the three angles in the triangle.

3) I have a mixture of 5p and 10p coins. Altogether I have twenty coins and they add up to a value of £1.65. Determine how many of each type of coin I have.

4) Two girls have 72 photographs of celebrities between them. One gives 11 to the other and finds that she now has half the number that her friend has.
Form an equation, letting n be the number of photographs one girl had at the **beginning**.
Hence find how many photographs each girl has **now**.

Chapter 3: SOLVING SIMULTANEOUS EQUATIONS

Simultaneous equations at GCSE/AS level usually consist of **a pair of equations** each in **two unknowns** which have to be solved **together**. There are a number of different ways of solving simultaneous equations but the most common are:

(i) the **substitution** method and (ii) the **elimination** method.

Example: Solve

$$3x + 2y = 8 \quad \dots ①$$
$$5x + y = 11 \quad \dots ②$$

Solution (by substitution method):

from ② $y = 11 - 5x \quad \dots ③$

sub ③ into ① $3x + 2(11 - 5x) = 8$

$$3x + 22 - 10x = 8$$

$$22 - 8 = 10x - 3x$$

$$14 = 7x$$

$$2 = x$$

sub into ③ $y = 11 - 5x = 11 - 5(2) = 11 - 10 = 1$

Therefore the solution is $x = 2$, $y = 1$.

Example: Solve

$$2x + 5y = 16 \quad \dots ①$$
$$3x - 4y = 1 \quad \dots ②$$

Solution (by elimination method):

① × 4 : $8x + 20y = 64 \quad \dots ③$

② × 5 : $15x - 20y = 5 \quad \dots ④$

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③+④ $23x = 69$

i.e. $x = 3$

sub into ① : $2(3) + 5y = 16$

$$6 + 5y = 16$$
$$5y = 10$$
$$y = 2$$

The solution is $x = 3$, $y = 2$

Exercise 3A

Solve the pairs of simultaneous equations in the following questions; remember to number/label your important equations and to refer to these number/labels when setting out your solutions.

1) $x + 2y = 7$
$3x + 2y = 9$

2) $x + 3y = 0$
$3x + 2y = -7$

3) $3x - 2y = 4$
$2x + 3y = -6$

4) $9x - 2y = 25$
$4x - 5y = 7$

5) $4a + 3b = 22$
$5a - 4b = 43$

6) 3 teas and 2 coffees cost £4.90, whereas
4 teas and 3 coffees would cost **an extra** £2.
Find the individual cost of each type of drink.

Chapter 4: FACTORISING EXPRESSIONS

In algebra, factorisation is the opposite to multiplying out brackets – we need the brackets to 'return'.

Simple Factorisation

Some algebraic expressions can be factorised by taking out all the common factors (i.e. the HCF) to the front of just one pair of brackets.

E1) $12x - 30 = 6(\ 2x - 5\ )$

E2) $6x^2 - 2xy = 2x(\ 3x - y\ )$

E3) $9x^3y^2 - 18x^2y = 9x^2y(\ xy - 2\ )$

E4) $9a^2b^3c + 6ab^2c^2 - 3a^3b^4 = 3ab^2(\ 3abc + 2c^2 - a^2b^2\ )$

E5) $3x(\ 2x - 1\ ) - 4(\ 2x - 1\ ) = (\ 2x - 1\ )(\ 3x - 4\ )$ … treating $(\ 2x - 1\ )$ as the HCF!

Exercise 4A

Factorise each of the following expressions:

1) $3x + xy$	**2)** $4x^2 - 2xy$
3) $pq^2 - p^2q$	**4)** $3pq - 9q^2$
5) $2x^3 - 6x^2$	**6)** $8a^5b^2 - 12a^3b^4$
7) $4mn^3 - 2m^3n + 6m^2n^2$	**8)** $5y(\ y - 1\ ) + 3(\ y - 1\ )$

Factorising quadratic expressions of the type $Ax^2 + Bx + C$

There are quite a few things to look out for when factorising quadratic expressions, as shown by the following different types of problems:

E1) $x^2 - 4x = x(x - 4)$

E2) $3x^2 + 6x = 3x(x + 2)$

… easy using simple factorisation, but this is very often forgotten!

E3) $x^2 + 5x + 6 = (x + 2)(x + 3)$

E4) $x^2 + 5x - 24 = (x + 8)(x - 3)$

E5) $x^2 - 14x + 24 = (x - 2)(x - 12)$

… should be relatively straightforward when the coefficient of x^2 is 1.

E6) $2x^2 + 11x + 5 = (2x + 1)(x + 5)$

E7) $3x^2 - x - 2 = (3x + 2)(x - 1)$

E8) $5x^2 - 16x + 3 = (5x - 1)(x - 3)$

… a little more thought is required here, but the fact that the coefficient of x^2 is a prime number helps to decide the opening terms in each bracket.

One of the most difficult types of quadratic expressions to factorise 'by inspection' is one in which both the coefficient of x^2 and the constant term have quite a few factors, e.g., $6x^2 + x - 12$ in which 6 has factors (±1, ±2, ±3 and ±6) and in which –12 has factors (±1, ±2, ±3, ±4, ±6 and ±12).

There is still an element of 'trial and error' involved but the systematic way of approaching this type of problem is shown below:

E9) Factorise $6x^2 + x - 12$

Thinking of $6x^2 + x - 12$ as $6x^2 + 1x - 12$ … we need to find a pair of numbers that multiply together to give $+6 \times -12 = -72$, and which add together to give +1.

The two required numbers are **–8** and **+9**.

So, rewriting, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} + \underbrace{9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets **must** be identical)

$= (3x - 4)(2x + 3)$

The difference of two squares: $\boldsymbol{x^2 - a^2 = (x - a)(x + a)}$

E10) $x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$

E11) $16x^2 - 25 = (4x)^2 - 5^2 = (4x - 5)(4x + 5)$

E12) $3x^2 - 48 = 3(x^2 - 16) = 3(x^2 - 4^2) = 3(x - 4)(x + 4)$

Exercise 4B

Factorise the following quadratic expressions

1) $x^2 + x$	**2)** $5x + 3x^2$
3) $6y^2 + 12y$	**4)** $x^2 + 8x + 15$
5) $x^2 - 7x + 10$	**6)** $m^2 - 2m - 8$
7) $2x^2 + 17x + 21$	**8)** $3x^2 - 5x - 2$
9) $7x^2 - 10x + 3$	**10)** $4x^2 + 16x + 15$
11) $6x^2 + 18x + 12$	**12)** $6x^2 - x - 1$
13) $12x^2 + 4x - 1$	**14)** $4p^2 + 9p - 9$
15) $2x^2 - 2$	**16)** $27a^2b^2 - 3c^2$

Chapter 5: CHANGING THE SUBJECT OF A FORMULA

Changing the subject of a formula is very similar to solving equations.

E1) Make x the subject of the formula $y = 4x + 3$.

Solution:

$$y = 4x + 3$$
$$y - 3 = 4x$$
$$\frac{y-3}{4} = x$$

So, $x = \dfrac{y-3}{4}$

E2) Make x the subject of $y = 2 - 5x$

Solution:

$$y = 2 - 5x$$
$$5x = 2 - y$$
$$x = \frac{2-y}{5}$$

E3) Make F the subject of the formula $C = \dfrac{5(F-32)}{9}$

Solution:

$$C = \frac{5(F-32)}{9}$$
$$9C = 5(F-32)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C+160}{5} = F$$

So, $F = \dfrac{9C+160}{5}$ or $F = \dfrac{9C}{5} + 32$

<u>Exercise 5A</u>

Make x the subject of each of these formulae:

1) $y = 7x - 1$

2) $y = \dfrac{x+5}{4}$

3) $4y = \dfrac{x}{3} - 2$

4) $y = \dfrac{4(3x-5)}{9}$

Rearranging equations involving squares and square roots

E4) Make x the subject of the formula $x^2 + y^2 = w^2$

Solution:

$$x^2 + y^2 = w^2$$
$$x^2 = w^2 - y^2$$
$$x = \pm\sqrt{w^2 - y^2}$$

E5) Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Solution:

$$t = \frac{1}{4}\sqrt{\frac{5a}{h}}$$
$$4t = \sqrt{\frac{5a}{h}}$$
$$16t^2 = \frac{5a}{h}$$
$$16t^2 h = 5a$$
$$\frac{16t^2 h}{5} = a$$
$$a = \frac{16t^2 h}{5}$$

Exercise 5B

Make t the subject of each of the following formulas:

1) $P = \frac{wt}{32r}$

2) $P = \frac{wt^2}{32r}$

3) $V = \frac{1}{3}\pi t^2 h$

4) $P = \sqrt{\frac{2t}{g}}$

5) $Pa = \frac{w(v-t)}{g}$

6) $r = a + bt^2$

More difficult examples - pay particular attention to the factorisation stage after which the required subject term now only appears **once** outside of a bracket.

E6) Make t the subject of the formula $a + xt = b - yt$

Solution:

$$a + xt = b - yt$$
$$xt + yt = b - a$$
$$t\,(x + y) = b - a$$
$$t = \frac{b - a}{x + y}$$

E7) Make W the subject of the formula $T - W = \dfrac{Wa}{2b}$

Solution:

$$T - W = \frac{Wa}{2b}$$
$$2bT - 2bW = Wa$$
$$2bT = Wa + 2bW$$
$$2bT = W(a + 2b)$$
$$\frac{2bT}{a + 2b} = W$$

or $\quad W = \dfrac{2bT}{a + 2b}$

Exercise 5C

Make x the subject of these formulae:

1) $ax + 3 = bx + c$

2) $3(x + a) = k(x - 2)$

3) $y = \dfrac{2x + 3}{5x - 2}$

4) $\dfrac{x}{a} = 1 + \dfrac{x}{b}$

Chapter 6: SOLVING QUADRATIC EQUATIONS

A quadratic equation (in x) has the form $ax^2 + bx + c = 0$ with $a \neq 0$.

The two main methods which are commonly used for solving quadratic equations are:
- factorising
- the quadratic formula

Note that not all quadratic equations can be easily solved by factorising.
However, the quadratic formula can always be used.

Method 1: Factorising

This method relies on the basic arithmetical fact that if the product of two factors is zero, then at least one of the two factors must be equal to zero.

i.e. if $\boldsymbol{PQ = 0}$ then **at least one** of $\boldsymbol{P}$ or $\boldsymbol{Q}$ must equal 0.

If necessary, rearrange the terms so that the term in x^2 (or y^2 … etc.) has a positive coefficient.

Example 1 : Solve $2x^2 + 5x - 3 = 0$

Solution: $(2x - 1)(x + 3) = 0$
So, either $2x - 1 = 0$ or $x + 3 = 0$

Therefore, the solutions (or **roots**) are $x = {}^1/_2$ or $x = -3$.

Example 2: Solve $x^2 + 9x = 0$

Solution: $x(x + 9) = 0$
So, either $x = 0$ or $(x + 9) = 0$
Giving $x = 0$ or $x = -9$

Example 3 Solve $11y - 6y^2 = 4$

Solution: $0 = 6y^2 - 11y + 4$ …which has **no** common factors (>1) in the RHS

Looking for two numbers which multiply to give $+6 \times +4 = +24$, and add to give -11 …
These numbers are **–3** and **–8**.

So,
$$\begin{aligned} 0 &= 6y^2 - 11y + 4 \\ &= 6y^2 - 3y - 8y + 4 \\ &= 3y(2y - 1) - 4(2y - 1) \\ 0 &= (2y - 1)(3y - 4) \qquad \therefore\ y = {}^1/_2 \ \text{ or } \ y = {}^4/_3 \end{aligned}$$

Method 2: Using the quadratic formula

Recall that the roots (i.e. solutions) of the quadratic equation $ax^2 + bx + c = 0$ (with $a \neq 0$) can be found using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 Solve the equation $x^2 - 2x - 7 = 0$

Solution: $x^2 - 2x - 7 = 0$ $\{\ a = 1,\ b = -2,\ c = -7\ \}$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-7)}}{2 \times 1} = \frac{2 \pm \sqrt{4 + 28}}{2} = \frac{2 \pm \sqrt{32}}{2}$$

$$= \frac{2 \pm \sqrt{16 \times 2}}{2} = \frac{2 \pm 4\sqrt{2}}{2}$$

$$x = 1 \pm 2\sqrt{2}$$

Therefore, the solutions are $x = 1 + 2\sqrt{2}$ or $x = 1 - 2\sqrt{2}$

Example 2 Solve the equation $6x + 1 = x(1 - 2x)$

Solution:

$$6x + 1 = x(1 - 2x)$$

$$6x + 1 = x - 2x^2$$

$$2x^2 + 5x + 1 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$x = \dfrac{-5 \pm \sqrt{17}}{4}$, sometimes seen as $x = \frac{1}{4}(-5 \pm \sqrt{17})$

Exercise 6A

1) Use factorisation to solve the following quadratic equations:

(*a*) $x^2 + 3x + 2 = 0$

(*b*) $x^2 - 3x - 4 = 0$

(*c*) $x^2 = 15 - 2x$

(*d*) $5x^2 + 3x - 2 = 0$

2) Determine the roots of the following equations:

(*a*) $x^2 + 3x = 0$

(*b*) $x^2 - 4x = 0$

(*c*) $4 - x^2 = 0$

(*d*) $\frac{2y^2}{3} - 5y = 0$

3) Solve the following equations either by factorising or by using the formula:

(*a*) $6x^2 - 5x - 4 = 0$

(*b*) $8x^2 - 24x + 10 = 0$

4) Use the quadratic formula to solve the equation $3x^2 - 2x - 4 = 0$

giving your solutions in the form $a \pm b\sqrt{13}$ where a and b are rational numbers.

Chapter 7: WORKING WITH INDICES

Basic rules of indices

y^4 means $y \times y \times y \times y$. 4 is called the **index** (plural: indices), **power** or **exponent** of y.

There are 3 basic rules of indices:

1) $a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$
2) $a^m \div a^n = a^{m-n}$ e.g. $3^8 \div 3^6 = 3^2$
3) $(a^m)^n = a^{mn}$ e.g. $(3^2)^5 = 3^{10}$

Further examples

$y^4 \times 5y^3 = 5y^7$

$4a^3 \times 6a^2 = 24a^5$ (multiply the coefficients and multiply the a terms)

$2c^2 \times \left(-3c^6\right) = -6c^8$ (multiply the coefficients and multiply the c terms)

$\frac{24d^7}{3d^2} = 8d^5$ (divide the coefficients and divide the d terms)

<u>Exercise 7A</u>

Simplify the following:

1) $b \times 5b^5$ (Remember that $b = b^1$)

2) $3c^2 \times 2c^5$

3) $b^2c \times bc^3$

4) $2n^6 \times (-6n^2)$

5) $8n^8 \div 2n^3$

6) $3d^{11} \div \frac{1}{2}d^9$

7) $(a^3)^2$

8) $(-d^4)^3$

More complex powers

Zero index:

Recall from GCSE that $a^0 = 1$. This result is true for any non-zero base a.

Therefore, $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Negative powers

A power of -1 corresponds to the **reciprocal** of a base, i.e. $a^{-1} = \frac{1}{a}$

Therefore $5^{-1} = \frac{1}{5}$

$0.25^{-1} = \frac{1}{0.25} = 4$

$\left(\frac{4}{5}\right)^{-1} = \frac{5}{4}$ (i.e. to find the reciprocal of a fraction just 'flip' it over)

This result can be extended to more general negative powers: $a^{-n} = \frac{1}{a^n}$.

This means:

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

$\left(\frac{1}{4}\right)^{-2} = \left(\left(\frac{1}{4}\right)^{-1}\right)^2 = \left(\frac{4}{1}\right)^2 = 16$

Fractional powers:

Fractional powers correspond to roots: $a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{4}} = \sqrt[4]{a}$

In general: $a^{\frac{1}{r}} = \sqrt[r]{a}$

Therefore: $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ $25^{\frac{1}{2}} = \sqrt{25} = 5$ $32^{\frac{1}{5}} = \sqrt[5]{32} = 2$

A more general fractional power can be dealt with in the following way:

$$a^{\frac{p}{r}} = \left(a^{\frac{1}{r}}\right)^p = \left(\sqrt[r]{a}\right)^p$$

So, $4^{\frac{3}{2}} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

$$\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\left(\frac{8}{27}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\left(\frac{25}{36}\right)^{-\frac{3}{2}} = \left(\frac{36}{25}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{36}{25}}\right)^3 = \left(\frac{6}{5}\right)^3 = \frac{216}{125}$$

Exercise 7B **no calculators allowed!**

Find the exact value in questions **1) – 12)** and simplify in questions **13) – 18)** :

1) $4^{\frac{1}{2}}$

2) $27^{\frac{1}{3}}$

3) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$

4) 5^{-2}

5) 18^{0}

6) 7^{-1}

7) $27^{\frac{2}{3}}$

8) $\left(\frac{2}{3}\right)^{-2}$

9) $8^{-\frac{2}{3}}$

10) $(0.04)^{\frac{1}{2}}$

11) $\left(\frac{27}{64}\right)^{\frac{2}{3}}$

12) $32^{-0.8}$

13) $2a^{\frac{1}{2}} \times 3a^{\frac{5}{2}}$

14) $x^{3} \times x^{-2}$

15) $\left(16x^{2}y^{4}\right)^{\frac{1}{2}}$

16) $\dfrac{\left(64x^{6}\right)^{\frac{1}{3}}}{\left(4x^{-2}\right)^{\frac{1}{2}}}$

Self Practice Test (45 min)

USE YOUR OWN FILE PAPER – there is not enough room on this page to do all your working!

You are NOT allowed to use a calculator.

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) Expand and simplify

(***a***) $(2x + 3)(2x - 1)$ (***b***) $(a + 3)^2$ (***c***) $4x(3x - 2) - x(2x + 5)$

2) Factorise

(***a***) $x^2 - 7x$ (***b***) $y^2 - 64$ (***c***) $2x^2 + 5x - 3$ (***d***) $6t^2 - 13t + 5$

3) Simplify

(***a***) $\dfrac{4x^3y}{8x^2y^3}$ (***b***) $\dfrac{3x+2}{3} + \dfrac{4x-1}{6}$

4) Solve the following equations

(***a***) $\dfrac{h-1}{4} + \dfrac{3h}{5} = 4$ (***b***) $x^2 - 8x = 0$ (***c***) $p^2 + 4p = 12$

5) Write each of the following as single powers of x and/or y

(***a***) $\dfrac{1}{x^4}$ (***b***) $(x^2y)^3$ (***c***) $\dfrac{x^5}{x^{-2}}$

6) Work out the values of the following, giving your answers as fractions

(***a***) 4^{-2} (***b***) 10^0 (***c***) $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

7) Solve the simultaneous equations

$$\begin{aligned} 3x - 5y &= -11 \\ 5x - 2y &= 7 \end{aligned}$$

8) Rearrange the following equations to make x the subject

(***a***) $v^2 = u^2 + 2ax$ (***b***) $T = \dfrac{a}{1-x}$ (***c***) $y = \dfrac{x+2}{x+1}$

9) Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form.

ANSWERS TO THE EXERCISES

CHAPTER 1 (Removing brackets)

Exercise 1A

1) $28x + 35$	**2)** $21 - 15x$	**3)** $4 - 7a$	**4)** $6y + 3y^2$
5) $-4x - 4$	**6)** $7x - 1$	**7)** $x^2 + 5x + 6$	**8)** $t^2 - 7t + 10$
9) $6x^2 + xy - 12y^2$	**10)** $4x^2 + 4x - 24$	**11)** $4y^2 - 1$	**12)** $12 + 17x - 5x^2$

Exercise 1B

1) $x^2 - 2x + 1$	**2)** $9x^2 + 30x + 25$	**3)** $49x^2 - 28x + 4$	**4)** $x^2 - 4$
5) $9x^2 - 1$	**6)** $25y^2 - 9$		

CHAPTER 2 (Solving linear equations)

Exercise 2A

1) 7	**2)** 3	**3)** 1½	**4)** 2	**5)** $-\frac{3}{5}$	**6)** $-\frac{7}{3}$
7) 5	**8)** 0	**9)** $-\frac{31}{13}$	**10)** -3	**11)** 2	**12)** 3

13) The 'equation' could lead to the statement '$0x = 0$' which is obviously **true** for **any** finite value of x.
14) The 'equation' could lead to the statement '$0x = 4$' which is **never true** for **any** finite value of x.

Exercise 2B

1) 2.4 **2)** 5 **3)** 1 **4)** $\frac{1}{2}$

Exercise 2C

1) 7 **2)** 15 **3)** $\frac{24}{7}$ **4)** $\frac{35}{3}$ **5)** 3 **6)** 2 **7)** $\frac{9}{5}$ **8)** 5

Exercise 2D

1) 18, 20, 22, 24	**2)** $A = 92°$, $B = 46°$ and $C = 42°$
3) seven 5p coins and thirteen 10p coins	**4)** 24, 48

CHAPTER 3 (Solving simultaneous equations)

Exercise 3A

1) $x = 1,\ y = 3$	**2)** $x = -3,\ y = 1$	**3)** $x = 0,\ y = -2$	**4)** $x = 3, y = 1$
5) $a = 7,\ b = -2$	**6)** $t = £0.90$, $c = £1.10$		

CHAPTER 4 (Factorising expressions)

Exercise 4A

1) $x(3 + y)$	**2)** $2x(2x - y)$	**3)** $pq(q - p)$	**4)** $3q(p - 3q)$
5) $2x^2(x - 3)$	**6)** $4a^3b^2(2a^2 - 3b^2)$	**7)** $2mn(2n^2 - m^2 + 3mn)$	**8)** $(y - 1)(5y + 3)$

Exercise 4B

1) $x(x + 1)$	**2)** $x(5 + 3x)$	**3)** $6y(y + 2)$	**4)** $(x + 3)(x + 5)$
5) $(x - 2)(x - 5)$	**6)** $(m - 4)(m + 2)$	**7)** $(2x + 3)(x + 7)$	**8)** $(3x + 1)(x - 2)$
9) $(7x - 3)(x - 1)$	**10)** $(2x + 3)(2x + 5)$	**11)** $6(x + 1)(x + 2)$	**12)** $(2x - 1)(3x + 1)$
13) $(6x - 1)(2x + 1)$	**14)** $(p + 3)(4p - 3)$	**15)** $2(x - 1)(x + 1)$	**16)** $3(3ab - c)(3ab + c)$

CHAPTER 5 (Changing the subject of a formula)

Exercise 5A

1) $x = \dfrac{y+1}{7}$	**2)** $x = 4y - 5$	**3)** $x = 12y + 6$	**4)** $x = \dfrac{9y+20}{12}$

Exercise 5B

1) $t = \dfrac{32Pr}{w}$	**2)** $t = \pm\sqrt{\dfrac{32Pr}{w}}$	**3)** $t = \pm\sqrt{\dfrac{3V}{\pi h}}$	**4)** $t = \dfrac{P^2 g}{2}$

5) $t = \dfrac{wv - Pag}{w}$ or $t = w - \dfrac{Pag}{w}$ **6)** $t = \pm\sqrt{\dfrac{r-a}{b}}$

Exercise 5C

1) $x = \dfrac{c-3}{a-b}$	**2)** $x = \dfrac{3a+2k}{k-3}$	**3)** $x = \dfrac{2y+3}{5y-2}$	**4)** $x = \dfrac{ab}{b-a}$

CHAPTER 6 (Solving quadratic equations)

Exercise 6A

1) (***a***) $-1, -2$ (***b***) $-1, 4$ (***c***) $-5, 3$ (***d***) $-1, \frac{2}{5}$

2) (***a***) $0, -3$ (***b***) $0, 4$ (***c***) $2, -2$ (***d***) $0, \frac{15}{2}$

3) (***a***) $\frac{4}{3}, -\frac{1}{2}$ (***b***) $\frac{5}{2}, \frac{1}{2}$

4) $x = \frac{1}{3} \pm \frac{1}{3}\sqrt{13}$

CHAPTER 7 (Working with indices)

Exercise 7A

1) $5b^6$ **2)** $6c^7$ **3)** b^3c^4 **4)** $-12n^8$ **5)** $4n^5$ **6)** $6d^2$ **7)** a^6 **8)** $-d^{12}$

Exercise 7B

1) 2 **2)** 3 **3)** $\frac{1}{3}$ **4)** $\frac{1}{25}$ **5)** 1 **6)** $\frac{1}{7}$ **7)** 9 **8)** $\frac{9}{4}$

9) $\frac{1}{4}$ **10)** 0.2 **11)** $\frac{9}{16}$ **12)** $\frac{1}{16}$ **13)** $6a^3$ **14)** x **15)** $4xy^2$ **16)** $2x^3$

ANSWERS TO THE SELF PRACTICE TEST

1) (***a***) $4x^2 + 4x - 3$ (***b***) $a^2 + 6a + 9$ (***c***) $10x^2 - 13x$

2) (***a***) $x(x - 7)$ (***b***) $(y + 8)(y - 8)$ (***c***) $(2x - 1)(x + 3)$ (***d***) $(3t - 5)(2t - 1)$

3) (***a***) $\frac{x}{2y^2}$ (***b***) $\frac{10x + 3}{6}$

4) (***a***) $h = 5$ (***b***) $x = 0$ or $x = 8$ (***c***) $p = 2$ or $p = -6$

5) (***a***) x^{-4} (***b***) x^6y^3 (***c***) x^7

6) (***a***) $\frac{1}{16}$ (***b***) 1 (***c***) $\frac{2}{3}$

7) $x = 3, y = 4$

8) (***a***) $x = \frac{v^2 - u^2}{2a}$ (***b***) $x = 1 - \frac{a}{T}$ or $x = \frac{T - a}{T}$ (***c***) $x = \frac{2 - y}{y - 1}$

9) $x = \frac{1 \pm \sqrt{21}}{10}$